Signal-to-Noise Ratio

Overview: This activity engages students with a hands-on activity and an online interactive to explore the Signal-to-Noise Ratio, a fundamental concept in spacecraft communication. The activity also includes a pencil-and-paper component that addresses relevant topics, such as proportions and ratios.

Target Grade Level: 6-8

Estimated Duration: 1 40-minute class session plus homework

Learning Goals: Students will be able to…

- Understand the terms "signal" and "noise" as they relate to spacecraft communication.
- Quantify noise using a given dataset.
- Calculate the signal-to-noise ratio.

Standards Addressed:

Benchmarks (AAAS, 1993)

The Nature of Technology, 3A: Technology and Science
The Physical Setting, 4F: Motion
The Designed World, 8D: Communication

National Science Education Standards (NRC, 1996)

Science in Personal and Social Perspectives: Science and Technology

Principles and Standards for School Mathematics (NCTM, 2000)

Number and Operations Standard
Measurement Standard
Connections Standard
Algebra Standard

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Teacher Background:

While some spacecraft return to Earth with valuable data as part of their cargo, all require some periodic remote communications as they travel. And for those spacecraft that do not return to Earth, the communication system is our only link to the data collected during its journey.

Not only do spacecraft transmit valuable data, but also spacecraft 'health' information is returned to Earth via these communication systems. It is important to know that the spacecraft's power systems, heating and cooling systems, and instruments are all operating as expected. And of course, signals must be sent to tell the spacecraft where to go or which instrument to operate and when via this system. Such course correction and data collection commands become even more critical as the spacecraft approaches its 'destination,' where course corrections become progressively finer and many of the science goals are to be achieved.

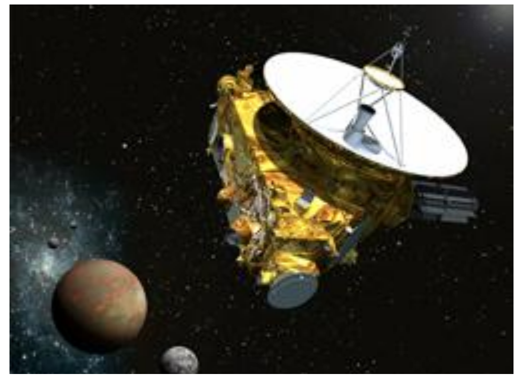


Figure 1. An artist's rendering of the New Horizons spacecraft as it approaches Pluto. The prominent 2.1-meter dish antenna is used to communicate with Earth from up to 7.5 billion kilometers away. (Image credit: JHUAPL/SwRI)

Each mission has its own telecommunications system design, but all use radio waves to transmit signals. Radio waves, like light waves, are part of the electromagnetic spectrum. As you can see in Figure 2, radio waves have long wavelengths, low frequencies, and—important for our ground-based communications—they penetrate Earth's atmosphere.

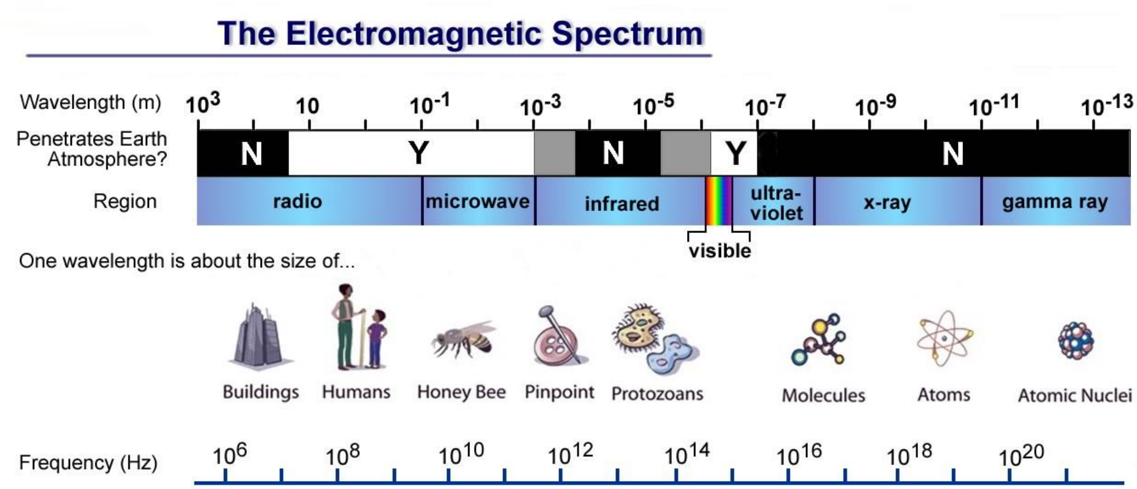


Figure 2. The electromagnetic spectrum. Notice radio waves penetrate Earth's atmosphere, have long wavelengths, and low frequencies. (Image courtesy: NASA).

Radio waves don't require as much energy for the spacecraft to produce as shorter wavelength electromagnetic waves do, which allows for more energy to power the instruments and other systems on a spacecraft. And unlike x-rays and shorter wavelengths, you don't have to protect yourself from them because they are harmless to humans. All of these characteristics make radio waves an ideal choice for carrying signals to and from spacecraft, as well as for carrying signals here on Earth for our TVs and radios.

Like all waves of the electromagnetic spectrum, radio waves travel at the speed of light. The speed of light in a vacuum is 299 792 458 meters per second, often approximated simply as 3 x 10^8 m/s. It is usually denoted by the symbol *c*, for the Latin *celeritas,* meaning "swiftness." Here on Earth, when you turn on the light switch the light seems to reach your eyes instantaneously. However, if you happen to be a mission operations flight controller sending an important command to a spacecraft—a signal that must travel many billions of kilometers—even the speed of light can seem slow. As the New Horizons spacecraft travels further away from Earth, its signals traveling at the speed of light take longer and longer to reach us.

The signal from the spacecraft is very weak by the time it reaches Earth, since its energy is spread over a wider and wider area as it travels outward from the transmitter. The signal from the spacecraft is not only extremely faint, it is embedded in a background of electromagnetic "noise." This noise is the incoherent background radiation produced by all other objects in the universe. It is always present in space, like static on your radio. Even while the New Horizons signal becomes fainter as the spacecraft gets farther away, the background noise remains at a roughly constant level. So the farther away the spacecraft, the more difficult it becomes to distinguish its signal from the noise. In addition, the communication equipment introduces its own noise.

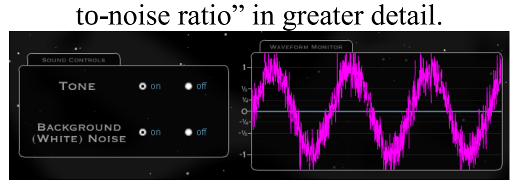


Figure 3. The online interactive at: http://patchyvalleyfog.com/signal_noise/ allows you to explore the concept of "signal-to-noise ratio" in greater detail.

With all of this noise, how can New Horizons communicate the data that it gathers on Pluto, its moons, and the Kuiper belt neighborhood back to us? The answer is that New Horizons must slow down its data transmission rate (the number of bits per second) as it gets further and further away. To be understood back here on Earth, New Horizons must "talk" more slowly as its signal becomes weaker.

This may seem peculiar. Why would it help to decrease the data transmission rate? In short, the answer is: for better signal detection. To understand, let's think of the data as a sequence of bits, or ones and zeros, that correspond to "signal on" and "signal off". The job of the radio receiver on Earth is to distinguish these two states of the signal from one another. That is, the receiver must decode the sequence of bits and pass the information along to a recording device.

The receiver does this by steadily making measurements—many each second—and averaging the results. But each time, of course, it is measuring not the signal alone, but the signal plus the noise. The averaging process preserves the signal (suppose for example it is "on" during the measurements) but it tends to reduce the effect of the noise. That's because a measurement of the noise is as likely to give a positive result as it is to give a negative one, and so the noise measurements can cancel each other out in the average. That cancellation is more and more effective the longer the averaging process goes on.

How much cancellation is needed? Basically, the averaging has to continue until the average of the noise is so low that "one" and "zero" can be distinguished from one another in the signal. The signal-to-noise ratio (SNR) compares the power level of the desired signal with that of the noise. A larger SNR indicates a stronger signal—that is, one that is easier to distinguish from the background. Therefore, the smaller the SNR, the more averaging that is needed. But increasing the averaging time means that the "ones" or "zeros" from the signal have to persist for a longer time too, or else they will cancel each other out as well. The result is that a long averaging time, which is needed to reduce the effect of the noise, requires a decreased data transmission rate. This can be likened to talking very slowly when trying to carry on a conversation in a crowded, noisy room.

The faintness of the signal in the presence of the background electromagnetic noise will force New Horizons to reduce its downlink rate for transmitting data to about 1000 bits per second. By comparison, if you connect to the internet with DSL or broadband cable modem, you "uplink" and "downlink" at a rate measured in "megabits" or millions of bits per second! At 1000 bits per second, it will take about 4 hours to downlink a picture of Pluto. But it will be well worth the wait!

Figure 4. The 70-meter (230 feet) antenna at the Goldstone Deep Space Communications Complex in the Mojave Desert, California. This is one of many radio antennae at Goldstone, which is one of the three facilities that make up NASA's Deep Space Network. (Image courtesy: NASA/JPL)

The faint signal can also be maximized with the help of very sophisticated and sensitive instruments, such as the Deep Space Network antennae. Since the background noise is constant, the more of the desired signal that the antenna can receive, the less averaging that needs to occur. This can be likened to adding a cupped hand next to your ear to better hear a conversation in a crowded, noisy room.

In this activity, students explore signals and noise as they relate to spacecraft communication and, more specifically, how they are compared and quantified with the signal-to-noise ratio (SNR). Before students can understand the SNR, however, they must first examine proportionality and ratios.

Two variables, x and y, are directly proportional if there is a non-zero constant, k, such that $y = k\,x$. In this case k is called the constant of proportionality, and is simply the ratio of y to x, so $k = y/x$. A graph of y as a function of x is a straight line passing through the origin with the slope equal to k (see graph on page 10 for an example). In this activity, the distance traveled by a signal is directly proportional to the time spent traveling, with the speed of light as the constant of proportionality.

We then explore an inversely proportional relationship, which behaves like a see-saw. Mathematically speaking, in this relationship one variable is directly proportional to the multiplicative inverse of the other, as in $y = k / x$. More simply, that means that when the magnitude of one variable goes up, the magnitude of the other goes down and their product, the constant of proportionality, k, is always the same. For example, as the distance from the source increases in the online interactive, the amplitude of the sine wave decreases. Note that the waveform monitor in the online interactive shows over-pressure (the difference between the actual pressure and the atmospheric pressure) versus time. We refer to the maximum deviation of the over-pressure as the amplitude or signal strength.

The graph of an inversely proportional relationship is a hyperbola, which isn't necessarily as easy for students to interpret as a straight line. Therefore, we also plot 1/(distance) versus amplitude, which is a straight line, to illustrate how variables can be manipulated to change the graph and perhaps make it easier to glean some information, such as the amplitude at a distance of 10 meters in this activity.

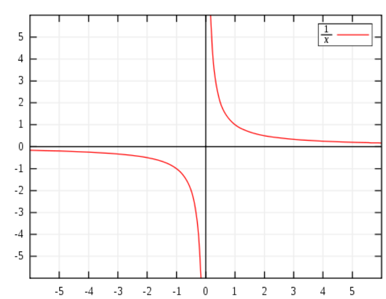


Figure 5. Graph of the reciprocal function, $y = 1/x$ for every x except 0. This is a simple example of a hyperbola.

Lastly, we use a ratio to represent the relationship between the signal and noise strength in spacecraft communication. A ratio simply compares two (or more) quantities. Ratios can be written as quotients or as quantities separated by a colon. As indicated previously, the signal-to-noise ratio compares the power level of the desired signal with that of the noise.

Materials:

- Copies of **Commands** sheets
  - sheets are to be cut in half, so print one quarter of a classroom set of the "sender" sheet and likewise for the "receiver" sheet
  - student pairs will receive one "sender" half-sheet and one "receiver" half-sheet
  - "sender" half-sheets can be used for multiple classes, but a new set of "receiver" half-sheets will need to be printed for each class
- A radio or a source for background noise (here is an option: http://patchyvalleyfog.com/signal_noise/ Use both the tone and the noise at full volume)
- Copies of **Signal-to-Noise Ratio** student background reading (1 set per student)
- Copies of **Signal-to-Noise Ratio student data sheets** (1 set per student)
- Access to a computer and projector or multiple computers with internet access
- OPTIONAL: Excel Student Data Sheet Worksheet (paper copies or digital version to manipulate)

Procedure:

Generally speaking…

What the teacher will do: Start by facilitating the warm-up activity, in which students talk with and without a radio/noise in the background from three distances (using the **Commands** sheets as their guide). Introduce the concepts of signals, noise, ratios and proportions, using information from the **Teacher Background** section if desired. Distribute pages 1-4 of the **Signal-to-Noise Ratio student data sheets** to students and divide class into small groups. Ask groups to complete Part I of the **student data sheets**, and then discuss results as a class. If possible, allow groups to move to a computer to explore the online **Signal-to-Noise Ratio interactive**. If multiple computers are not available, project the interactive from your computer and begin exploring the "tone" portion of the interactive. Groups should then complete Part II of the **Signal-to-Noise ratio student data sheets** and then Part III of the **student data sheets** after exploring the "noise" portion of the interactive. Allow students plenty of time to complete Parts II and III before distributing pages 5 and 6 (and Part IV) of the **student data sheets**; you might need to assign these remaining pages as homework (it is not necessary to complete them in groups). Distribute the **Signal-to-Noise** student background reading along with Part IV of the data sheets. Conclude with a brief discussion of the signal-to-noise ratio and how scientists and engineers maximize the signal to and from a spacecraft. The student data sheets may be collected and used for assessment if desired.

What the students will do: Begin with an activity in which pairs of students try to send/receive commands using the **Commands** sheets with and without a radio/noise in the background from three different distances. Then students will be introduced to the concepts of signals, noise, ratios and proportions before breaking into groups. They will complete Part I of the **Signal-to-Noise Ratio student data sheets** and discuss the results as a class. Then they will explore the online interactive, **Signal-to-Noise Ratio**, and complete Parts II and III of the **student data sheets**. The teacher will then distribute pages 5 and 6 of the **student data sheets** along with the **Signal-to-Noise Ratio** student background reading so students can complete the remainder of Part III and Part IV either in class or as homework. Finally, students will participate in a class discussion about the signal-to-noise ratio as it relates to spacecraft communication.

Advance Preparation

- Make copies of handouts, as indicated in **Materials** section
- Cut **Commands** sheets in half along dashed line
- Select a radio station from a radio or the computer. If unavailable, use the tone and noise turned "on" from here: http://patchyvalleyfog.com/signal_noise/

- Arrange for access to multiple computers with internet access, if possible

In-class Procedure

Warm up

1. Begin with a brief warm-up activity to introduce "signals" and "noise" as follows:
2. Divide class into pairs. Give each pair one "sender" **Commands** half-sheet and one "receiver" **Commands** half-sheet. Be sure that the "receiver" does not see the "sender's" **Commands** sheet.
3. Ask each pair to stand about 1-meter apart and facing each other.
4. Explain that the students are to "send" (speak) a signal and "receive" (hear/record) a signal without background noise and then with background noise (i.e. a radio or noise that you turn on/off). You should prompt them through the commands so that command 1 is "sent" (spoken) without background noise, then command 2 is "sent" after you have turned on the noise, etc. Alternate so that the first command at each distance is without noise and the second is "sent" after you have turned on the noise source. The "sender" reads a command from his/her sheet and the "receiver" records what he/she hears in the appropriate space. Be sure you are always turning the noise to the same volume. Indicate that there are two rules:
    a. Students CANNOT raise their voices or try to talk louder during the activity
    b. Students CANNOT lean closer
5. Then both students should take two large steps backward (away from each other) and again, send/receive a signal with and without background noise and again obeying the two rules. You will have to turn the radio on for the "with noise" trial to the same volume each time.
6. Repeat again after both students take two more steps away from each other (while obeying the rules). Again, you need to turn the radio on/off.
7. Ask groups to compare the command that was spoken by the "sender" with what was recorded by the "receiver". How do the commands compare with and without noise? How does distance change the communication?
8. As a class, discuss sending and receiving the commands with and without background noise and at increasing distances. What were some of the techniques they used to try to send/received as the distance increased (without raising their voices or leaning closer!) (Desired answers: **talk more slowly, cup hand near ear**, enunciate very clearly, read lips)

Activity

9. This activity assumes at least a basic understanding of ratios and proportions. If you haven't already done so, introduce these topics along with "signals" and "noise" as they relate to spacecraft communication. You might wish to use information contained in the **Teacher Background** section, above.
10. Explain to students that you will be exploring the "signal-to-noise ratio" using an online interactive and student data sheets as you distribute only pages 1-4 of the **Signal-to-Noise Ratio** student data sheet. (Note: these are pages 10-13 of this complete module).
11. Divide the class into small groups of 2-4 students. If possible, adjust the size of the groups so that each group can work at a computer during parts II-IV of the activity/student data sheets.
12. Ask student groups to complete Part I of the student data sheets. Allow them enough time to finish (about 5 minutes) before discussing the results as a class. This Part explores a directly proportional relationship (time and distance with the speed of light as the constant of proportionality).
13. For Parts II and III of the **Signal-to-Noise Ratio** student data sheets, students will need access to computers/the internet to explore the Signal-to-Noise Ratio online interactive at this URL: http://patchyvalleyfog.com/signal_noise/ If multiple computers are not available, you could explore the interactive as a class with one computer and a projector, however this is not

ideal. Allow students plenty of time to complete these two sections. **NOTE:** the last two questions in Part III, questions 4 and 5, will not be completed now to facilitate the inquiry exercise in question number 3 of Part III. If time permits, discuss as a class some of the ideas proposed in number 3 before moving on.

14. After students have completed Part II and all but the last two questions in Part III, distribute the last two pages of the **Signal-to-Noise student data sheets** (Part III, questions 4 and 5, and Part IV—pages 14-15 of this module) and the **Signal-to-Noise** student background reading (page 22 of this module). These questions do not require the online interactive, nor do they require group work but one requires internet access. The background reading will help them to answer these questions. As time permits, students can complete these readings and remaining pages in their groups, but some will likely need to be assigned as homework or completed during the following day.
15. **NOTES for Part III, questions 4 and 5**: OPTIONAL: You may also wish to provide them with the optional Excel spreadsheet to help them calculate the noise values for these questions. Since calculations can be tedious and are not the goal of this activity, this spreadsheet is provided. You have three options:
    a. Don't use the spreadsheet; have students calculate noise values using averages and square roots "by hand."
    b. Provide the digital version of the spreadsheet and ask students to calculate the noise values using Excel. DOWNLOAD SPREADSHEET (http://pluto.jhuapl.edu/common/content/pdfs/snr_student_data_sheet_workbook.xlsx) They need to fill in the pink boxes with formulas as follows (you can give them these formulas if desired):
        i. Part III, question 4: insert formula (between quotes/not including quotes) in pink cell E30 (average of the absolute values) "=AVERAGE(C28,G28)"
        ii. Part III, question 5: insert the following formulas (between quotes) in the appropriate cells:
            1. In cell D70 (sum of squared noise values) "=SUM(D37:D68)"
            2. In cell D71 (Average of squared noise values) "=AVERAGE(D37:D68)"
            3. In cell D72 (square root of average) "=SQRT(D71)"
    c. Fill in the above formulas for them, save changes and print out these sheets for them to use along with the Signal-to-Noise Ratio student data sheet. If you feel students are not prepared to calculate averages and square roots, this is your best choice.
16. **NOTES for Part IV**:, For questions 1 and 2, students can use the interactive to find the signal values at the requested distances, or they can use the table, graph, and what they learned in Part II. Question 4, which requires reading the **Signal-to-Noise** student background section, could serve as a summative assessment of desired.
17. Finally, it is important to discuss what they have learned and brainstormed. The teacher **ANSWERS: Signal-to-Noise Ratio student data sheets** will help guide your discussion of the individual questions in the student data sheets.

Extensions and Adaptations:

- Accompany this activity with the "Earth Calling…" activity, a hands-on activity exploring spacecraft radio communication concepts, including the speed of light and the time-delay for signals sent to and from spacecraft. "Earth Calling…" is available here: http://pluto.jhuapl.edu/common/content/pdfs/snr_student_data_sheet_workbook.xlsx

- For hearing impaired students, the hands-on introduction to signals and noise can be supplemented with the online interactive, "Signal-to-Noise Ratio," since it provides a graphical equivalent of signals and noise from various distances. (http://patchyvalleyfog.com/signal_noise/)

- The student background reading mentions "cupping your ear" to better hear something in the presence of background noise. This "sound cone" could be used to represent the large antenna found on spacecraft and Earth, such as those used in the Deep Space Network. Here is an activity in which students use a cone constructed of paper to explore how this works: http://spaceplace.nasa.gov/en/kids/tmodact.shtml

Resources:

- The New Horizons website, with a discussion of data transmission processes: http://pluto.jhuapl.edu/science/soc.php
- Cramer, K. & Post, T. (1993, May). Connecting Research to Teaching Proportional Reasoning. *Mathematics Teacher*, 86(5), 404-407. http://cehd.umn.edu/rationalnumberproject/93_2.html
- The NASA Jet Propulsion Laboratory "Basics of Space Flight" has a chapter on Telecommunications that may be useful background reading material for the teacher: http://www2.jpl.nasa.gov/basics/bsf10-1.php
- The NASA Space Place has a short video about spacecraft communication that may be a useful resource for the students and teachers alike: http://spaceplace.nasa.gov/en/kids/st5xband/st5xband.shtml
- This activity exploring binary notation and spacecraft communication may be useful for background reading or to further explore this topic: http://spaceplace.nasa.gov/en/educators/dsn_signal_mod_web.pdf

Standards:

Benchmarks

3. The Nature of Technology

    A. Technology and Science, 6-8th grades:

    - Technology is essential to science for such purposes as access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.

4. The Physical Setting

    F. Motion, 6-8th grades:

    - There are a great variety of electromagnetic waves: radio waves, microwaves, infrared waves, visible light, ultraviolet rays, X-rays, and gamma rays. These wavelengths vary from radio waves, the longest, to gamma rays, the shortest.

8. The Designed World

    D. Communication, 6-8th grades:

    - Errors can occur in coding, transmitting, or decoding information, and some means of checking for accuracy is needed. Repeating the message is a frequently used method.
    - Information can be carried by many media, including sound, light, and objects. In the 1900s, the ability to code information as electric currents in wires, electromagnetic waves in space, and light in glass fibers has made communication millions of times faster than mail or sound.

Principles and Standards for School Mathematics (NCTM, 2000)

Number and Operations Standard for Grades 6-8

Understand numbers, ways of representing numbers, relationships among numbers, and number systems:

- understand and use ratios and proportions to represent quantitative relationships

Compute fluently and make reasonable estimates

- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios

Measurement Standard for Grades 6-8

Apply appropriate techniques, tools, and formulas to determine measurements

- solve problems involving scale factors, using ratio and proportion

Connections Standard for Grades 6-8

- recognize and apply mathematics in contexts outside of mathematics.

Algebra Standard for Grades 6-8

- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

National Science Education Standards (NRC, 1996)

Science in Personal and Social Perspectives Standard

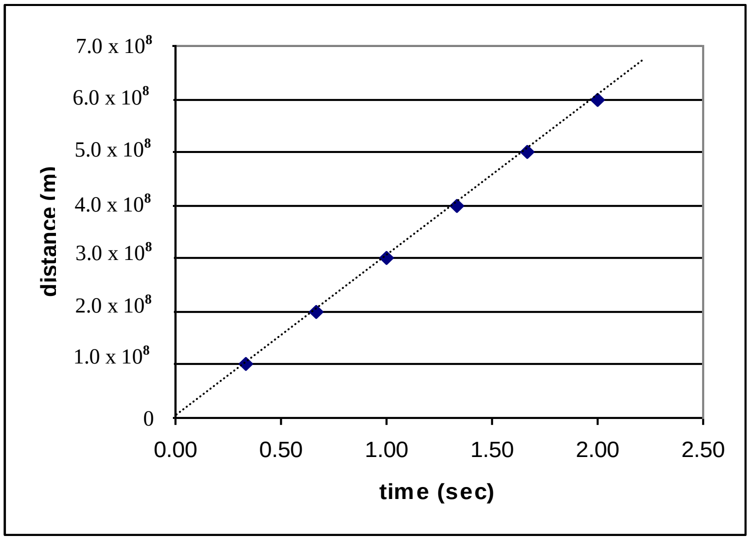
Science and Technology, Levels 5-8

- Abilities of technological design
- Understanding about science and technology

Signal-to-Noise Ratio
Student Data Sheets

Part I. When we send a signal to a spacecraft, it is useful to know how long it will take to arrive at the spacecraft. It may be obvious that the further a signal has to travel, the longer it will take to arrive. However, there is much more to learn about how these quantities are related by looking closely at the data and graph of the time it took for a signal to travel several different distances.

time (sec)	distance (m)
0.33	1.0×10^8
0.67	2.0×10^8
1.00	3.0×10^8
1.33	4.0×10^8
1.67	5.0×10^8
2.00	6.0×10^8



1. If the signal traveled to the Moon (about 4.0×10^8 meters) in 1.33 seconds, how long would it take to travel 8.0×10^8 meters?

2. Would a signal traveling from Earth to the Moon and back arrive at Earth a) before b) after or c) at the same time as the signal that traveled 8.0×10^8 meters?

3. Notice the graph of the distance vs. time is a straight line that passes through the origin (0,0). This means that the distance (d) traveled and the time (t) are *directly proportional*, and the equation for such a line looks like:

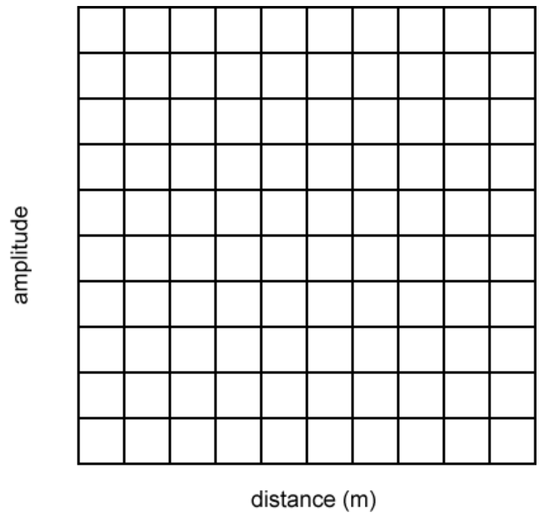
$$d = k\, t$$

Calculate ***k***, or the constant of proportionality, for this data set:

4. What characteristic of the graph above does ***k*** describe?

Part II. Navigate to the online interactive exploring the signal-to-noise ratio at: http://patchyvalleyfog.com/signal_noise/. Try turning on the "tone" and observing what happens to the amplitude of the wave in the waveform monitor when you move the speaker along the distance scale.

distance (m)	**amplitude**
1	
	1/2
4	
8	



1. Complete the data table and graph, above, using the Signal-to-Noise Ratio interactive (Note: **turn on just the "tone" for Part II**).

2. Describe the pattern you see of how the amplitude changes with distance. How is this similar to or different from the "distance versus time" data from Part I?

3. Are both quantities (distance and amplitude) increasing together, which would mean they are directly related, or is one increasing as the other is decreasing, which would mean they are inversely related?

4. Look closely at the data in the data table above. Describe the relationship between the distance and amplitude numbers.

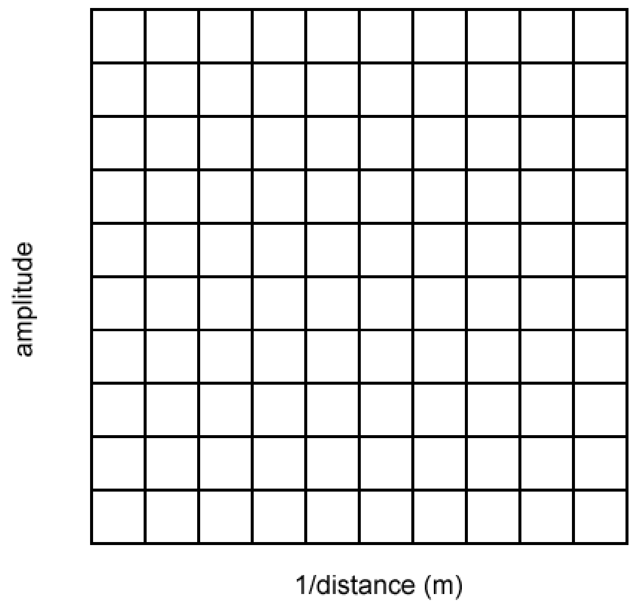
5. Since the amplitude decreases as the distance increases, we say these two quantities are inversely related. But more specifically, we see in the answer to #4 above that these quantities are inversely *proportional*, such that

amp. = k/dist. Where k is a non-zero constant

What is the value of ***k*** for this data set?

6. Given the relationship between amplitude and distance is inversely proportional, it is convenient to graph the data as (1/distance) versus amplitude. Fill in the data table and graph below:

1/distance (m)	**amplitude**
	1
	1/2
1/4	
1/8	



7. How does the graph of “distance” vs. “amplitude” from #1 (Part II) compare with the graph of “1/distance” vs. “amplitude” in #6, above?

8. Using the graph in #6 and what you know about the relationship between distance and amplitude, what would the amplitude be at a distance of 10 meters?

Part III. Navigate to an online interactive exploring the signal-to-noise ratio at: http://patchyvalleyfog.com/signal_noise/. Now try turning on the "noise" and observing what happens to the amplitude of the wave in the waveform monitor when you move the speaker along the distance scale.

1. How does the curve for "noise" appear different from the curve for "tone" that you observed in Part II?

2. In Part II, you were able to complete a data table and graph for the "tone" at various distances vs. amplitudes. If **just** the "**noise**" is turned on, can you easily complete a similar data table, like below?

time	amplitude
1	
2	
4	
8	

3. You were probably not able to easily "quantify" or assign a number to the "noise" level at different distances in #2, above. Spend the next 5 minutes and discuss as a group how you might quantify noise. Record your ideas and calculations here:

4. One way to quantify the amount of noise is to separate the data into those noise values that are positive (above the x-axis) and those that are negative (below the x-axis). You can take the average of the positive values and the average of the negative values and then average the absolute values of those two numbers together to find a single value for "noise". Try this technique with the following data set:

Time	noise
9	0.35
9.5	-1.87
10	0.53
10.5	1.36
11	-1.18
11.5	-1.20
12	1.81
12.5	1.61
13	-1.81
13.5	2.29
14	0.31
14.5	0.59
15	-0.28
15.5	-0.21
16	0.62
16.5	-0.55

time	noise
1	-0.34
1.5	0.01
2	-0.07
2.5	0.07
3	-2.01
3.5	1.20
4	-0.54
4.5	-0.32
5	-0.56
5.5	1.15
6	1.42
6.5	-0.93
7	0.02
7.5	1.01
8	-0.74
8.5	-0.88

5. Another common method scientists use to quantify noise is accomplished by squaring each noise value and averaging the results. The squared values are always positive, even if the noise values are negative. The square root of the average of the squares (the "root mean square") is then a form of quantification for the noise. Try to calculate the "root mean square noise" using this method with the dataset from #4, above.

Part IV. We have explored signals and noise and how they are calculated. These values are important for spacecraft communications; they are used to calculate the Signal-to-Noise Ratio. This ratio helps the engineers distinguish the signal they are trying to receive from the background noise that is always present in space.

1. Calculate the signal-to-noise ratio for the online interactive using the signal value at 2 meters and the noise value calculated in Part III, number 5 (above).

2. This time calculate the signal-to-noise ratio for the online interactive using the signal value at 6 meters and the noise value calculated in Part III, number 5 (above).

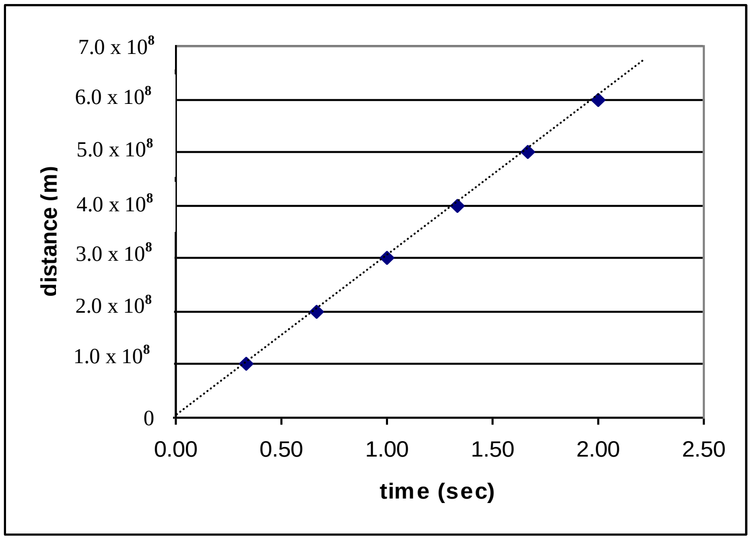
3. Looking at the above two calculations, which do you think is the "better" signal-to-noise ratio if you were an engineer wishing to glean important information from the signal and why?

4. After reading the Signal-to-Noise Ratio material provided, describe in your own words the problem and brainstorm at least two ideas of how engineers might be able to maximize the amount of information received from a signal, even when a spacecraft is very far away. Record your ideas below or on a separate page if necessary.

Signal-to-Noise Ratio student data sheets
ANSWER KEY FOR TEACHERS

Part I. When we send a signal to a spacecraft, it is useful to know how long it will take to arrive at the spacecraft. It may be obvious that the further a signal has to travel, the longer it will take to arrive. However, there is much more to learn about how these quantities are related by looking closely at the data and graph of the time it took for a signal to travel several different distances.

time (sec)	distance (m)
0.33	1.0×10^8
0.67	2.0×10^8
1.00	3.0×10^8
1.33	4.0×10^8
1.67	5.0×10^8
2.00	6.0×10^8



1. If the signal traveled to the Moon (about 4.0×10^8 meters) in 1.33 seconds, how long would it take to travel 8.0×10^8 meters?

$t = d / k$
$t = (8.0 \times 10^8 \text{ m})/(3.0\ 10^8 \text{ m/s}) = 2.66$ sec
Note: students can also use proportional reasoning to solve the problem with the same result:
$t_1/d_1 = t_2/d_2$
1.33 sec/4.0×10^8 m = $t_2/8.0 \times 10^8$ m
$t_2 = 2.66$ sec

2. Would a signal traveling from Earth to the Moon and back arrive at Earth a) before b) after or c) at the same time as the signal that traveled 8.0×10^8 meters?

c. at the same time as the signal that traveled 8.0×10^8

3. Notice the graph of the distance vs. time is a straight line that passes through the origin (0,0). This means that the distance (d) traveled and the time (t) are *directly proportional,* and the equation for such a line looks like:

$$d = k\,t$$

Calculate ***k***, or the constant of proportionality, for this data set:

1.0×10^8 m = (0.33 sec) k
$k = 1.0 \times 10^8$ m/ 0.33 sec = 3.0×10^8, to this level of accuracy.
Note: The actual speed of light in a vacuum is 299 792 458 m/s

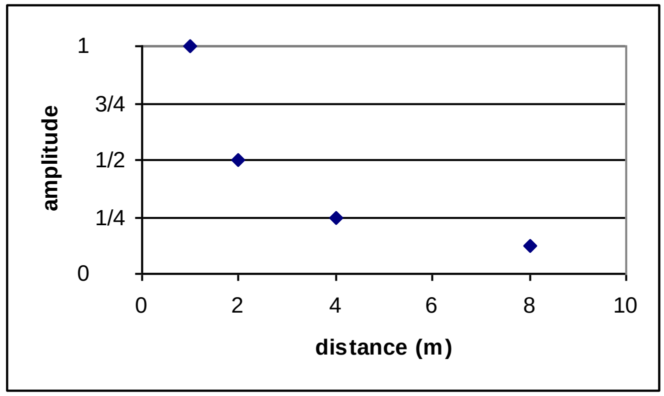
4. What characteristic of the graph above does ***k*** describe?

The slope of the line

Part II. Navigate to an online interactive exploring the signal-to-noise ratio at: http://patchyvalleyfog.com/signal_noise/. Try turning on the "tone" and observing what happens to the amplitude of the wave in the waveform monitor when you move the speaker along the distance scale.

1. Complete the data table and graph, below, using the Signal-to-Noise Ratio interactive (Note: turn on just the "tone" for Part II).

distance (m)	amplitude
1	1
2	½
4	¼
8	1/8



Note: students will likely complete the graph by hand, but it should look similar to the one above.

2. Describe the pattern you see of how the amplitude changes with distance. How is this similar to or different from the "distance versus time" data from Part I?

The graph of this data is a curve and does not pass through the origin, whereas the "distance versus time" data is a straight line and it does pass through the origin.

3. Are both quantities (distance and amplitude) increasing together, which would mean they are directly related, or is one increasing as the other is decreasing, which would mean they are inversely related?

One is increasing as the other is decreasing; as the distance increases the amplitude decreases, which means they are inversely related.

4. Look closely at the data in the data table above. Describe the relationship between the distance and amplitude numbers.

The numbers are related so that the amplitude is the same as 1/distance; the distance and the amplitude are inversely proportional; if you removed the "1/" from the amplitude numbers, they are the same as the distance.

5. Since the amplitude decreases as the distance increases, we say these two quantities are inversely related. But more specifically, we see in the answer to #4 above that these quantities are inversely *proportional*, such that

amp. = k/dist. Where k is a non-zero constant

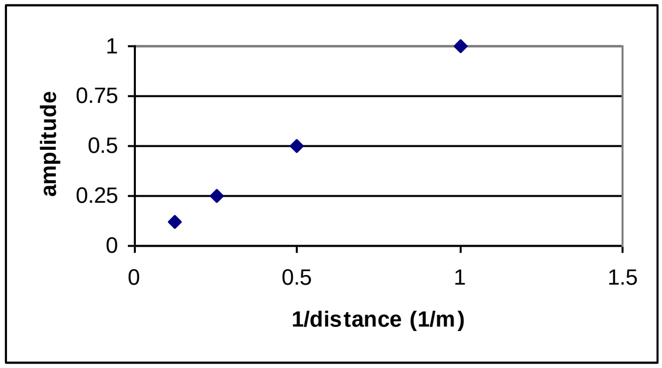
What is the value of ***k*** for this data set?

For this data set $k = 1$
For example: $\frac{1}{2} = k/2$

$k = 1$

6. Given the relationship between amplitude and distance is inversely proportional, it is convenient to graph the data as (1/distance) versus amplitude. Fill in the data table and graph below:

1/distance (m)	amplitude
1	1
1/2	1/2
1/4	1/4
1/8	1/8



Note: students will likely complete the graph by hand, but it should look similar to the one above.

7. How does the graph of "distance vs. amplitude" from #1 (Part II) compare with the graph of "1/distance vs. amplitude" in #6, above?

In #1 the graph is a curve and as the distance increases the amplitude decreases. However, this graph is a straight line and as 1/distance increases so does the amplitude.

8. Using the graph in #6 and what you know about the relationship between distance and amplitude, what would the amplitude be at a distance of 10 meters?

At a distance of 10 meters, the amplitude would be 1/10.

Part III. Navigate to an online interactive exploring the signal-to-noise ratio at: http://patchyvalleyfog.com/signal_noise/. Now try turning on the "noise" and observing what happens to the amplitude of the wave in the waveform monitor when you move the speaker along the distance scale.

1. How does the curve for "noise" appear different from the curve for "tone" that you observed in Part II?

The curve of noise versus time is bumpy or jagged, unlike the smooth line for tone. The noise curve has many more deviations above and below the x-axis. The noise curve deviations are irregular; unlike the tone curve, the noise curve does not follow a pattern, but seems to be very random.

2. In Part II, you were able to complete a data table and graph for the "tone" at various distances vs. amplitudes. If **just** the "**noise**" is turned on, can you easily complete a similar data table, like below?

time	amplitude
1	
2	
4	
8	

No. It would be very difficult to try to complete the above data table. The noise is very random.

3. You were probably not able to easily "quantify" or assign a number to the "noise" level at different distances in #2, above. Spend the next 5 minutes and discuss as a group how you might quantify noise. Record your ideas and calculations here:

Answers will vary for this question. This is designed to be an inquiry exercise to get students thinking about possible solutions. The students will not have access to the next two questions (#s 4 and 5), but you can look ahead and see two different ways scientists quantify noise.

4. One way to quantify the amount of noise is to separate the data into those noise values that are positive (above the x-axis) and those that are negative (below the x-axis). You can take the average of the positive values and the average of the negative values and then average the absolute values of those two numbers together to find a single value for "noise". Try this technique with the following data set:

time	**noise**
1	-0.34
1.5	0.01
2	-0.07
2.5	0.07
3	-2.01
3.5	1.20
4	-0.54
4.5	-0.32
5	-0.56
5.5	1.15
6	1.42
6.5	-0.93
7	0.02
7.5	1.01
8	-0.74
8.5	-0.88

time	**noise**
9	0.35
9.5	-1.87
10	0.53
10.5	1.36
11	-1.18
11.5	-1.20
12	1.81
12.5	1.61
13	-1.81
13.5	2.29
14	0.31
14.5	0.59
15	-0.28
15.5	-0.21
16	0.62
16.5	-0.55

ANSWER:

positive noise values	negative noise values
0.01	-0.34
0.07	-0.07
1.20	-2.01
1.15	-0.54
1.42	-0.32
0.02	-0.56
1.01	-0.93
0.35	-0.74
0.53	-0.88
1.36	-1.87
1.81	-1.18
1.61	-1.20

Average of positive deviations = 0.90
Average of negative deviations = -0.84

Average of the absolute value of the averages, or noise value for the dataset using this method = 1.74/2 = 0.87

2.29	-1.81
0.31	-0.28
0.59	-0.21
0.62	-0.55

5. Another common method scientists use to quantify noise is accomplished by squaring each noise value and averaging the results. The squared values are always positive, even if the noise values are negative. The square root of the average of the squares (the "root mean square") is then a form of quantification for the noise. Try to calculate the "root mean square noise" using this method with the dataset from #4, above.

ANSWER:

time	noise	noise squared
1	-0.34	0.12
1.5	0.01	0.00
2	-0.07	0.01
2.5	0.07	0.00
3	-2.01	4.03
3.5	1.20	1.43
4	-0.54	0.29
4.5	-0.32	0.10
5	-0.56	0.31
5.5	1.15	1.32
6	1.42	2.02
6.5	-0.93	0.86
7	0.02	0.00
7.5	1.01	1.03
8	-0.74	0.54
8.5	-0.88	0.77

time	noise	noise squared
9	0.35	0.13
9.5	-1.87	3.51
10	0.53	0.28
10.5	1.36	1.86
11	-1.18	1.40
11.5	-1.20	1.45
12	1.81	3.28
12.5	1.61	2.59
13	-1.81	3.28
13.5	2.29	5.23
14	0.31	0.10
14.5	0.59	0.35
15	-0.28	0.08
15.5	-0.21	0.04
16	0.62	0.38
16.5	-0.55	0.31

Sum of the squared noise values = 37.09
Average of the squared noise values = 37.09/32 = 1.16
Square root of the average of the squared noise values, or the noise value using this method = **1.08**

Part IV. We have explored signals and noise and how they are calculated. These values are important for spacecraft communications; they are used to calculate the Signal-to-Noise Ratio. This ratio helps the engineers distinguish the signal they are trying to receive from the background noise that is always present in space.

1. Calculate the signal-to-noise ratio for the online interactive using the signal value at 2 meters and the noise value calculated in Part III, number 5 (above).

Signal-to-noise ratio: 0.5:1.08 = 0.46

2. This time calculate the signal-to-noise ratio for the online interactive using the signal value at 6 meters and the noise value calculated in Part III, number 5 (above).

Signal-to-noise ratio: 0.17:1.08 = 0.16

3. Looking at the above two calculations, which do you think is the "better" signal-to-noise ratio if you were an engineer wishing to glean important information from the signal and why?

A larger SNR indicates a stronger signal—that is, one that is easier to distinguish from the background. So the SNR for #1, from 2 meters away, is a "better" or more desirable SNR than the one for #2 at 6 meters away. This makes sense; the signal strength is greater closer to the source and therefore easier to distinguish from the constant background noise.

4. Using the resources provided as background reading material, brainstorm at least two ideas of how engineers might be able to maximize the amount of information received in a signal, even when a spacecraft is very far away. Record your ideas and how they help obtain information here:

Answers will vary. For discussion purposes you may want to note ideas like:

- Use very sensitive antennae or transmitters and receivers so that even over great distances a signal can be recorded.
- Students may be familiar with tuning a radio—with careful tuning to the frequency of the signal one can filter out much noise
- Have the spacecraft "talk slowly" when it is very far away; if the signal is sent very slowly it is easier to distinguish from the constant background noise. This is sort of like when you are in a crowded, noisy room trying to maintain a conversation—if you talk slowly you are better able to communicate despite the noise.

The Signal-to-Noise Ratio

What is a signal?

When you listen to the radio or talk on a cell phone, do you ever wonder how the information you are hearing travels to the radio or phone? The answer is: radio waves!

Radio waves, like light waves, are part of the electromagnetic spectrum. As you can see in Figure 1, radio waves have long wavelengths, low frequencies, and—important for our ground-based communications—they penetrate Earth's atmosphere. And unlike x-rays and shorter wavelengths, you don't have to protect yourself from them because they are harmless to humans. All of these characteristics make radio waves an ideal choice for carrying signals for our radios and cell phones as well as for carrying signals to and from spacecraft.

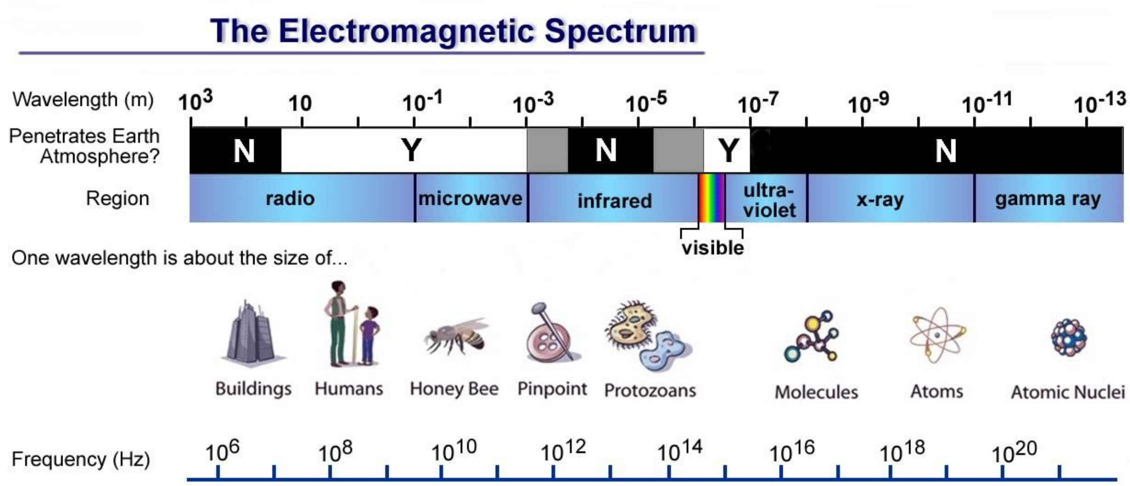


Figure 1. The electromagnetic spectrum. Notice radio waves penetrate Earth's atmosphere, have long wavelengths, and low frequencies. (Image courtesy: NASA).

Each mission has its own telecommunications system design, but all use radio waves to transmit (that is, send and receive) signals to and from the spacecraft. Not only do spacecraft transmit valuable data, but also spacecraft "health" information is returned to Earth via these communication systems. It is important to know that the spacecraft's power systems, heating and cooling systems, and instruments are all operating as expected. And of course, signals must be sent to tell the spacecraft where to go or which instrument to operate and when via this system. Such course correction and data collection commands become even more critical as the spacecraft approaches its targeted destination, since this is where many of the goals of the mission are to be achieved.

Like all waves of the electromagnetic spectrum, radio waves travel at the speed of light. The speed of light in a vacuum is 299 792 458 meters per second, often approximated simply as 3×10^8 m/s. It is usually denoted by the symbol *c*, for the Latin *celeritas,* meaning "swiftness." Here on Earth, when you turn on the light switch the light seems to reach your eyes instantaneously. However, if you happen to be a mission operations flight controller sending an important command to a spacecraft—a signal that must travel many billions of kilometers—even the speed of light can seem slow. As the New Horizons spacecraft travels further away from Earth, its signals traveling at the speed of light take longer and longer to reach us.

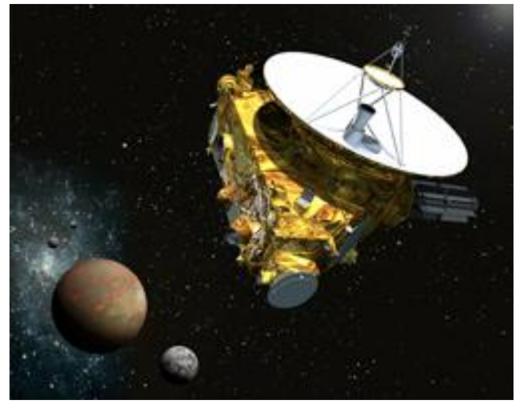


Figure 2. An artist's rendering of the New Horizons spacecraft as it approaches Pluto. The prominent 2.1-meter dish antenna is used to communicate with Earth from up to 7.5 billion kilometers away. (Image credit: JHUAPL/SwRI)

The signal from the spacecraft is also very weak by the time it reaches Earth, since it is spread over a wider and wider area as it travels outward from the transmitter. You can demonstrate this yourself using a flashlight and a wall. Stand very close to the wall and point your flashlight at the wall. Do you see a small circle of light on the wall (about the same size as the light-end of the flashlight)? Now take a few steps away from the wall. Has the "signal" or light from the flashlight spread out, making the circle on the wall larger, but also not as bright (weaker)? Take another few steps away from the wall and try this again. What do you see? Eventually, if you are far enough from the wall, the signal from the flashlight will be so weak you will no longer be able to see a circle of light on the wall.

What is noise?

The signal from the spacecraft is not only extremely faint, it is mixed with "noise." This noise comes from all objects in the universe and is always present in space. Even while the New Horizons signal becomes fainter as the spacecraft gets farther away, the background noise remains at a roughly constant level. So as the signal becomes fainter it becomes more difficult to distinguish it from the noise. Using our flashlight example from above, try the same experiment but have someone switch the lights "on" and "off" at the increasing distances. Imagine the light in the room as "noise"—with the lights on the circle on the wall from the flashlight is harder to see than with the lights "off."

The signal-to-noise ratio (SNR) compares how much of the desired signal we receive (the "power" of the signal) with the noise we receive. A larger SNR indicates a stronger signal—that is, one that is easier to distinguish from the background. So as the spacecraft travels farther away, the SNR gets smaller.

You have personal experience with noise obscuring a signal if you have ever heard static on the radio. But do you know what noise "looks" like in data?

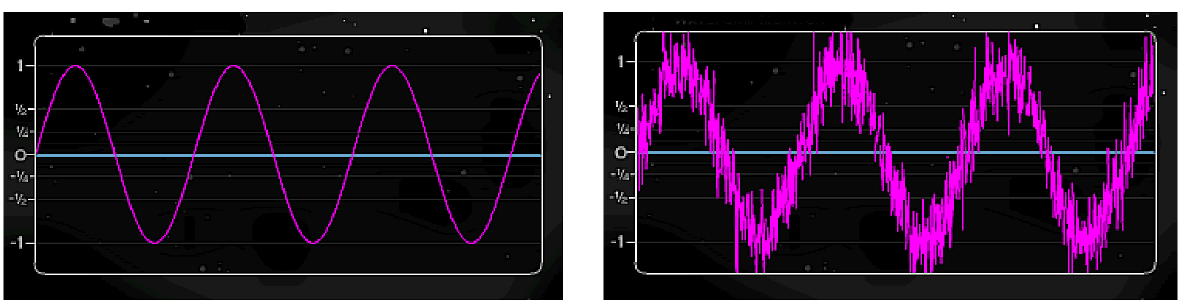


Figure 3. The image at left is a pure tone or signal with an amplitude of 1. The image at right is the same tone, however now there is a constant background noise along with the signal. (See these for yourself at the online interactive: http://patchyvalleyfog.com/signal_noise/)

Notice in Figure 3 how the noise along with the signal causes small deviations above and below the signal curve. It actually looks noisy, doesn't it? Now let's see what it would look like if we moved farther away from the signal, just as a spacecraft often moves farther from us as it sends signals.

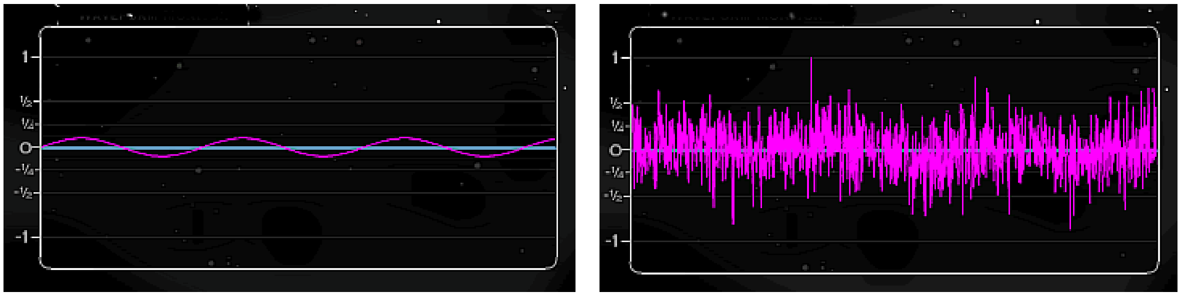


Figure 4. Again, the image at left is a pure tone signal, however the receiver (ear) has moved farther from the source of the tone, thereby reducing the amplitude of the sine curve as compared with Figure 3. The image at right is the same tone, and the receiver is at the same distance, but now in the presence of the constant background noise.

Notice that in this Figure 4, the signal is weaker since the receiver is farther from the source or tone. When the constant background noise is turned on it is very difficult to see the desired signal. When you compare the noisy image (right) in figure 3 with that in figure 4 you can see how it becomes difficult to distinguish the noise from the desired signal as the distance between the source (i.e. spacecraft) and the receiver (i.e. antenna here on Earth) increases.

You probably have some personal experience with noise. Have you ever tried to have a conversation at a concert, near a loud TV, or in a noisy room? A "noisy room" is basically the same as "noisy data." The desired signal is the information your friend is trying to tell you, and the noise is…well, the noise! If you are trying to talk to someone in the presence of a lot of background noise, there are a few things you might do to better hear them. You might find yourself leaning closer, cupping your hand next to your ear to increase the "size" of your ear (the "receiver"), talking very loudly or very slowly to help distinguish their voice from the background noise.

How can we increase the SNR?

Figure 5. The 70-meter (230 feet) antenna at the Goldstone Deep Space Communications Complex in the Mojave Desert, California. This is one of many radio antennae at Goldstone, which is one of the three facilities that make up NASA's Deep Space Network. (Image courtesy: NASA/JPL)

One method we can use to help distinguish the signal from the noise in the data we receive from the New Horizons spacecraft is to have the spacecraft "talk more slowly" as it travels further from Earth. The spacecraft must slow down its data transmission rate (the number of bits per second) as it gets further and further away. The faintness of the signal in the presence of the background electromagnetic noise will force New Horizons to reduce its downlink rate (much like a "download" rate on your own computer) for transmitting data to about 1000 bits per second. By comparison, if you connect to the internet with DSL or broadband cable modem, you "uplink" and "downlink" at a rate measured in "megabits" or millions of bits per second! At 1000 bits per second, it will take about 4 hours to downlink a picture of Pluto. But it will be well worth the wait!

The faint signal can also be maximized with the help of very sophisticated and sensitive instruments, such as the Deep Space Network antennae. Since the background noise is constant, the more of the desired signal that the antenna can receive the better. This can be likened to adding a cupped hand next to your ear to better hear a conversation in a crowded, noisy room.

"Talking slowly" and "cupping our ear" help to increase how much of the signal we receive, but we are usually forced to further adjust the incoming data to improve the SNR. We do this using a method of averaging. Let's take another look at that graph of the distant tone and noise data.

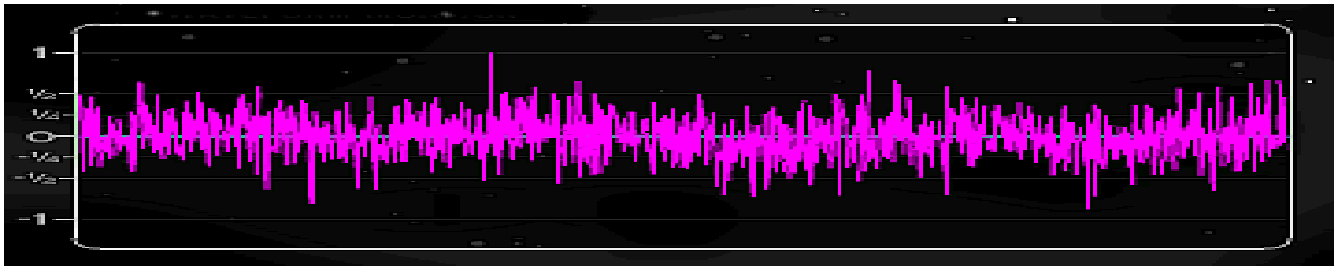


Figure 6. The image of the desired tone signal from far away and constant background noise has been elongated along the x-axis so you can see the noise peaks more clearly.

Notice how the noise adds closely-spaced peaks to the desired signal, and the peaks are both positive and negative. In fact, a measurement of the noise is as likely to give a positive result as it is to give a negative one, and so the noise measurements can cancel each other out in the average. That cancellation is more and more effective the longer the averaging process goes on.

Therefore, the smaller the SNR, the more averaging that is needed. The tricky part for the scientists and engineers to figure out is exactly how much averaging is needed so that they minimize the noise yet preserve the desired signal.

Commands sheet: SENDER

Directions: You (the "sender") will send commands to "the receiver" (your partner) from three distances, both with and without background noise. Your partner should write down what he/she hears. However, as you try to communicate your command to the receiver there are two rules you must follow:

1. You CANNOT raise your voice or try to talk louder. You must try to speak at the same volume for each distance and with our without the background noise.
2. You CANNOT lean toward your partner, and nor can they lean toward you.

Commands:

1. Turn on the number two camera.
2. Turn on the first laser altimeter.
   (take two steps away from each other)
3. Turn off the left thruster.
4. Turn off the number one camera.
   (take two steps away from each other)
5. Turn on the right thruster.
6. Turn off the laser altimeter.

Commands sheet: SENDER

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Commands:

1. Turn on the first laser altimeter.
2. Turn on the right thruster.
   (take two steps away from each other)
3. Turn off the laser altimeter.
4. Turn off the left thruster.
   (take two steps away from each other)
5. Turn off the number one camera.
6. Turn on the number two camera.

Commands sheet: RECEIVER

Directions: You (“the receiver”) will record the commands you hear from the “sender” (your partner) using the spaces provided below. You will be receiving commands from three distances, both with and without background noise. However, as you try to listen for the command from your partner there are two rules:

1. You CANNOT lean toward your partner, and nor can they lean toward you.
2. Your partner CANNOT raise his/her voice despite the changing distance and the presence or absence of background noise.

Commands you received (heard):

1.
2.
3.
4.
5.
6.

Commands sheet: RECEIVER

Directions: You (“the receiver”) will record the commands you hear from the “sender” (your partner) using the spaces provided below. You will be receiving commands from three distances, both with and without background noise. However, as you try to listen for the command from your partner there are two rules:

1. You CANNOT lean toward your partner, and nor can they lean toward you.
2. Your partner CANNOT raise his/her voice despite the changing distance and the presence or absence of background noise.

Commands you received (heard):

1.
2.
3.
4.
5.
6.