

Materials:

- copy of the Degree Ruler (4 m) Teacher Page
- copies of Earth and Pluto Diagrams (3 copies of each)
- tape/material to affix ruler and planet pages to the board or wall
- meter stick (2 would be helpful)
- masking tape to mark distances on the floor, and three short pieces per student
- marker to write distances on tape
- classroom set of Angles in the Sky student reading (1 per student)
- classroom set of Student Data Sheets (1 per student)

Procedure:

Generally speaking...

What the teacher will do: The teacher will copy handouts and diagrams to post. He/she will arrange the classroom with adequate space near the three stations to accommodate groups of students. At each station he/she will place diagrams on the wall/board and tape on the floor at indicated distances from the diagrams. The teacher will divide the students in pairs, and during the activity he/she will rotate between the three stations to answer questions.

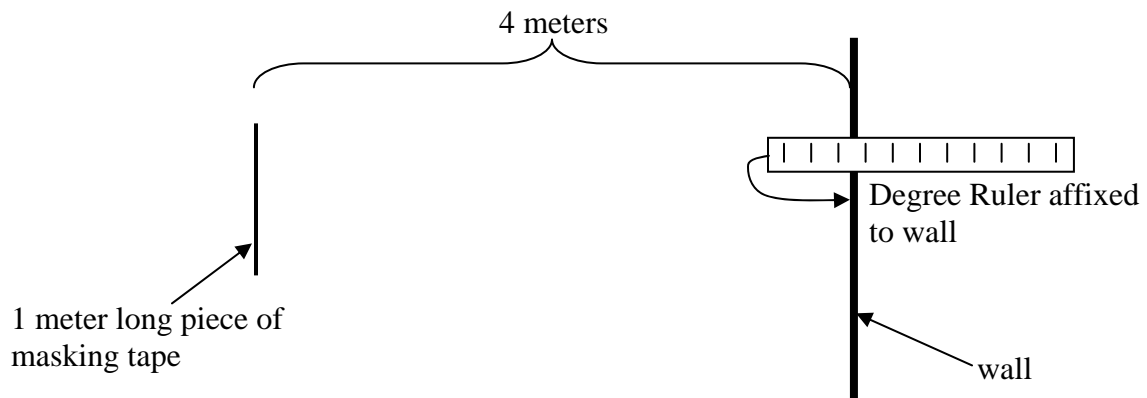
What the students will do: Within each pair, the students will take turns as data recorder and data collector or measurer. They will rotate through the three stations, collecting different data at each station. After finishing the “Collecting Data” section they will return to their desks to complete the “Observations” and “Calculations” sections of their Student Data Sheets. The final question on the Student Data Sheet can be assigned for homework if the moon is visible in the evening sky.

Advance Preparation

1. Introduce the students to the concepts of actual, apparent, and angular width. Use material from the Background section if desired. Prior to the activity assign students to read Angles in the Sky.
2. *Setting up the classroom:* At each station you will affix a diagram or two (depending on the station; see individual station instructions below) to the wall/board. You will need to clear a space in front of the diagram(s) to a distance of about 5 m, and wide enough to allow several students to stand side-by-side in front of it. You may want to enlist the help of students in the task of moving desks after introducing the activity.

Station 1: Affix the Degree Ruler Teacher Page on the board or wall. Place a long (about 1 meter) piece of masking tape on the floor parallel with the wall and wide enough for a few students to stand side-by-side at a distance of 4 meters from the Degree Ruler. This station will be used for step 1 of the Student Data Sheet.

For example, from above:



Station 2: Affix one set of Earth and Pluto diagrams side-by-side. Place long pieces of masking tape on the floor in front of the diagrams at distances of 1 m, 2 m, and 4 m (parallel to wall and long enough to allow a few students to stand side-by-side). Again, allow a distance of 5 m from the planet diagrams and enough elbow room to accommodate several students at once. This station will be used for step 2 of the Student Data Sheet. (Similar to diagram above, but with three pieces of tape at 1 m, 2 m, and 4 m from the wall).

Station 3: Affix one set of Earth and Pluto diagrams side-by-side. Leave a meter stick and masking tape at this station. This station will be used for step 3 of the Student Data Sheet.

In-class Procedure

1. Explain to students that they will be doing a hands-on activity to explore the idea of angular width. Provide each student with a Student Data Sheet. Divide class into teams of two. One student will serve as the data recorder, while the other is determining the width of his or her finger, thumb, and fist; then roles will be reversed. This teamwork will be continued throughout the activity.
2. Demonstrate the proper technique in determining the angular width of your finger, thumb, and fist in degrees. Stand at the 4 m mark and extend your arm out **straight in front of you** at about eye level. Align your finger with the left side of the Degree Ruler and (closing one eye) determine how many degrees your finger is wide. **REMEMBER: Keep your arm straight!** Remind students to do the same! Continue in the same manner with thumb and fist. You may want to talk about standards of measurement. For example, should the thumb be inside the fist or outside when measuring the width of a fist. Allow class to come to an agreement or establish the 'standards of measurement'.
3. Allow students to cycle through the stations. The most efficient technique will be to have several students' measure at once, while their partner data recorders stand

out of the way. Measurers can tell the data recorder the results as they go, and then the partners can switch. They do not have to go through the stations in order, since they will not be using the degree calibration until the 'observations' and 'calculations' sections. So you can divide the class into three roughly equal groups, and each group can start at a different station.

Station 1: Students stand at the piece of tape and determine the angular width of their fingers, thumbs and fists. As pairs, one student measures while the other records the measurements on the Student Data Sheet and then the roles reverse.

Station 2: Students stand at the piece of tape and determine the angular width of the Earth and Pluto diagrams in fingers, thumbs and fists. Answers can be in fractions of fingers, thumbs or fists, such as $\frac{1}{2}$ a finger. Lines on the planet diagrams are placeholders to make measurements like "2 fingers" easier. Again, students cooperate in pairs, trading roles of measurer and recorder.

Station 3: Students must choose one planet or the other and use three pieces of tape to record on the floor the distance at which the planet is 1 finger wide, 2 fingers wide, and 3 fingers wide. They must then use a meter stick to determine the distance from the planet diagram to their three pieces of tape on the floor. They will measure and record these three distances on their Student Data Sheet for both members of the pair.

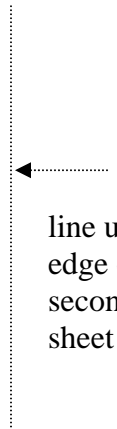
START



1°



2°



line up
edge of
second
sheet here

3°



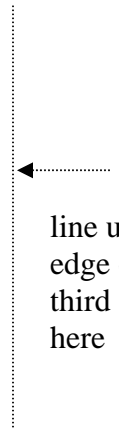
4°



5°



6°



line up
edge of
third sheet
here

7°



8°



9°



←
line up
edge of
fourth
sheet here

10°



11°



12°



13°



line up
edge of
fifth sheet
here

A vertical dotted line with a horizontal arrow pointing left towards the 13° line.

14°



15°



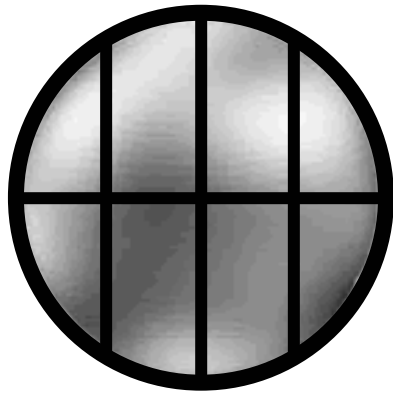
16°



Earth



Pluto





Angles in the Sky

Think for a moment about the size of the Sun and the size of the Moon as you see them in the sky. Do they appear to be similar in size?

Here are their actual sizes:

Sun's diameter = 1,391,000 kilometers (862,400 miles)

Moon's diameter = 3,476 kilometers (2,160 miles)

And just for reference:

Earth's diameter = 12,742 kilometers (7,900 miles)

Why do you think the Sun and Moon appear to be similar in size, and yet their actual sizes are so dramatically different? To uncover the answer, we need to learn more about the difference between an object's apparent size and its actual size.

Here on Earth we can often determine the size of objects or the distance between objects using rulers or measuring tapes. Astronomers do not have that luxury, since they can rarely visit the object they wish to measure. But they do have a systematic way of describing how large an object appears to be—that is, its apparent size. Angular measurements are used to indicate the apparent size of an object in the sky. Angular measurements employ the degree ($^{\circ}$) as a fundamental unit, with 360° in a full circle (Figure 1).

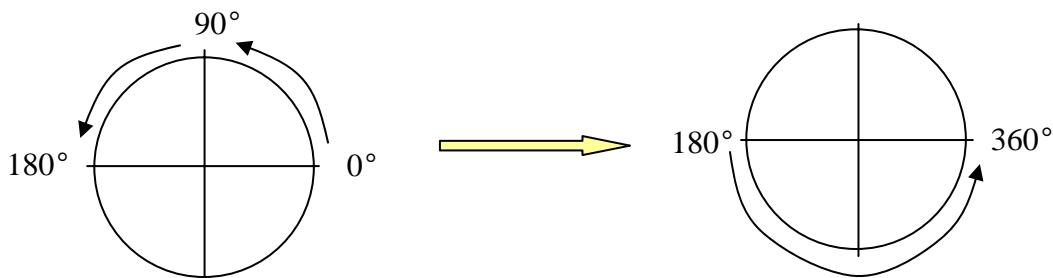


Figure 1. Half a circle is 180° , which is like the horizon if you are standing on the surface of the Earth. There are 360° in a full circle.

The apparent size of an object depends both on the object's actual size and its distance from the observer. As an example, the Sun and Moon appear to be similar in size; the angular diameter of each is about $\frac{1}{2}^{\circ}$ (Fig. 2). Yet the Sun's actual diameter is hundreds of times larger than the Moon's. How can their apparent sizes be the same? Here are their distances from Earth:

Average distance to the Sun = 149,597,890 km (92,955,820 miles)

Average distance to the Moon = 384,403 km (238,857 miles)

The Sun is much, much farther from the Earth than is the Moon, and so even though it is also much larger than the Moon it appears to us to be about the same size.

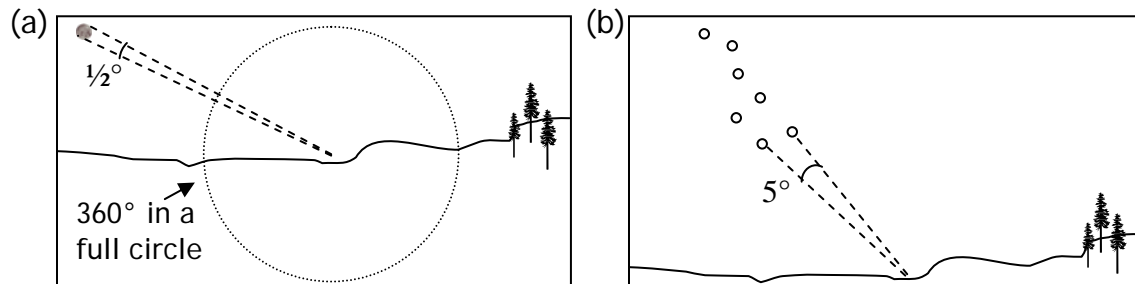
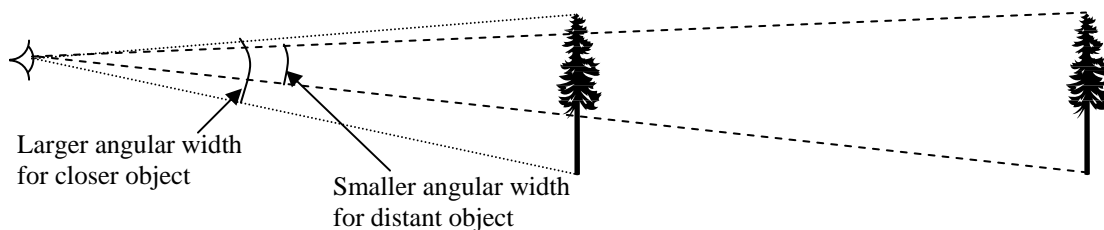


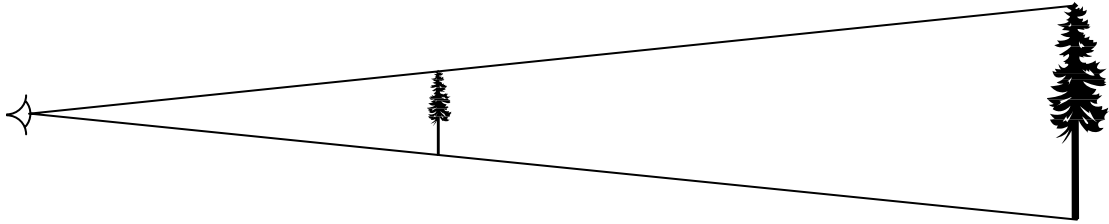
Figure 2. An example of the system of measurement based on angles in degrees, with 360 degrees in a full circle. (a) The full moon, when observed from Earth, has an angular diameter of about $\frac{1}{2}^\circ$. (b) Two stars indicated in the Big Dipper are separated by an angular distance of 5° .

Here are two ways to think about the relationship between apparent size and actual size:

3. For an object of a given actual size, its apparent (angular) size is smaller the further away from you it is. For example, if you are standing next to a mature tree it seems large, whereas if you are standing 100 yards from it, the tree appears smaller. If you move to 200 yards from it, the tree will appear smaller still. Finally you will reach a distance where the tree appears so small it cannot be distinguished from the surrounding landscape.

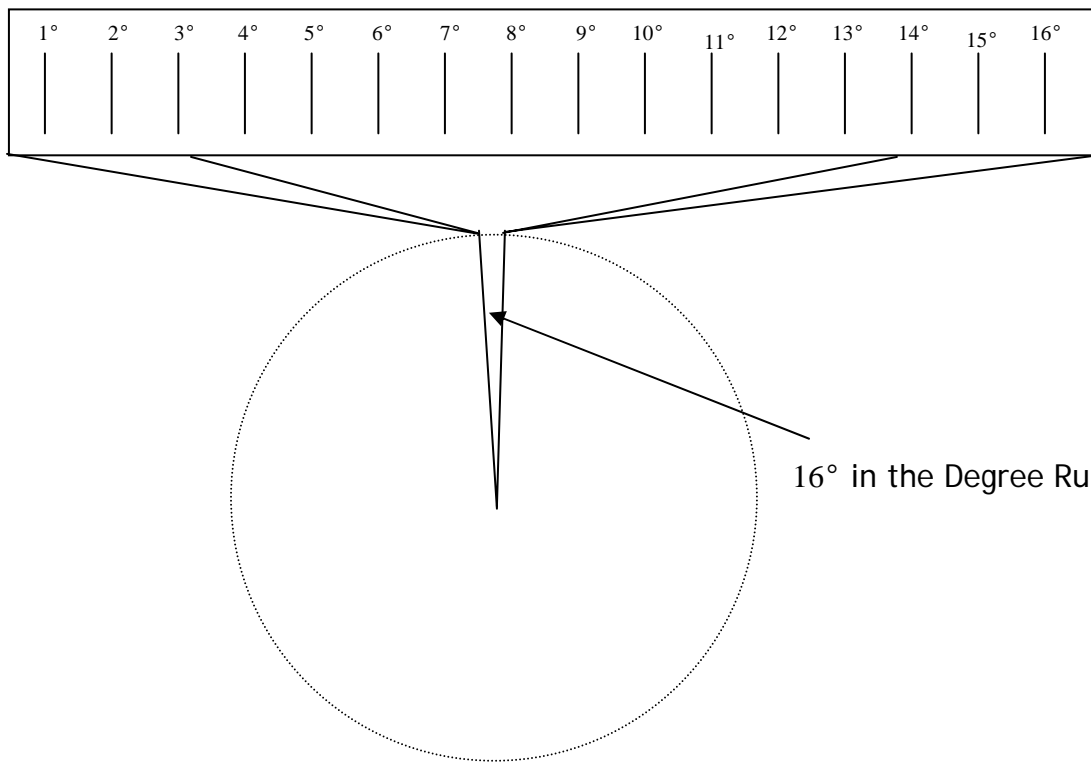


4. If two objects have the same angular size, the actual size of the more distant object is greater than that of the nearby object. For example, the tip of your pinky finger has an angular width of about $\frac{1}{2}^\circ$ if your arm is extended straight in front of you and held at eye level. Likewise, the full Moon has an angular diameter of about $\frac{1}{2}^\circ$. Therefore, they both have the same angular size. Yet your pinky finger is about 1 cm wide and a few feet from you, while the Moon is 3,476 km in diameter and about 384,000 km away.



You will be exploring apparent or angular size and distance in much greater detail in a hands-on activity. In the activity you will use a Degree Ruler that is based on a full circle with 360° . However, the Degree Ruler you use will only be a small portion of the 360° .

Here is what the Degree Ruler will look like. It is enlarged, but you can see it is only a small part (16° !) of a whole circle:



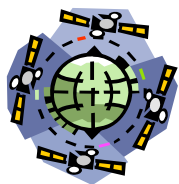
Activity: Appearances Can Be Deceiving!

Name:

Today you are going to explore a measuring tool that you always carry with you. It is the very same tool you use to write, and the same tool you use to drink a glass of water. But did you know you could use your hand as a measuring tool?

Here is how it works: First notice that your teacher has placed a Degree Ruler and some planet diagrams on the board or wall. Also, on the floor are pieces of tape indicating the distances from the objects on the board or wall. You are going to determine the angular width of your finger, thumb, and fist using the Degree Ruler and the angular width of the diagrams of Pluto and the Earth while standing at these pieces of tape.

Proper technique is VERY important! Always hold your arm out straight in front of you at eye level and extend either your pointer finger, thumb or make a fist. You will be working with a partner. One of you will take a measurement while the other records the data, and then you will switch roles. You will visit all three stations before going to the ‘Observations’ and ‘Calculations’ sections.



Data Collection

Step 1. At **STATION 1** you will determine the width or angular width of your finger, thumb, and fist in degrees, and record the data below. Begin by extending your arm out straight in front of you at eye level. Extend your index finger (“pointer finger”) straight up. Without bending your arm, line up the left edge of your finger with the left side of the Degree Ruler (marked with “START HERE”). Determine how many degrees your finger covers—that is, how wide it is in degrees.

Distance from the Degree Ruler: _____ meters

	Finger	Thumb	Fist
Angle measure:			

Step 2. At **STATION 2** you will see a diagram of Earth and a diagram of Pluto on the wall, with three pieces of tape on the floor labeled with the distance from the diagram. The planet diagrams use the same relative scale for the size of the planets. This means that it looks as if you were standing the same distance from each of them. While standing at each piece of tape, you should now measure the angular width of each planet using your finger, thumb, and fist. Remember to hold your arm out straight in front of you! Record your measurements below:

Distance from the planet diagrams: _____ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

Distance from the planet diagrams: _____ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

Distance from the planet diagrams: _____ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

Step 3. At **STATION 3** you will see the Earth and Pluto planet diagrams on the wall. Choose one of the planets, and determine how far away from the diagram you must stand so that it is exactly one finger wide, exactly two fingers wide, and exactly three fingers wide. Mark the distances on the floor with a piece of tape, and measure them using a meter stick. Record your distances below:

Planet chosen: _____

1 finger wide = _____ meters

2 fingers wide = _____ meters

3 fingers wide = _____ meters



Observations

- In step 2 you measured the angular widths of the planets in units of ‘number of fingers, thumbs, and fists’. Compare your values with your partners. Are they exactly the same or are they different? Also compare your values with the values of another group. Are all of the values the same or do they differ?

a) If your measurements from step 2 are different from your partners or from another group's measurements, does it mean the size of the planets changed? What is different?

b) If people have different answers for step 2, is one wrong and another right? Why or why not?

5. In step 2 you measured the angular width of Earth from three distances. In the first row of the table below, record the three different distances from which Earth was measured in order of increasing distance. In the second row of the table, record the angular width of Earth (measured in fingers) at the corresponding distance.

	Closest to Earth	Middle	Farthest from Earth
Distances, in order of closest to farthest:			
Angular width, in fingers, corresponding to the distance in the box above it:			

a) In the table above, as the distance increases (i.e. as you move farther from the diagram of Earth) what happens to the angular width of Earth, as measured in 'fingers' ?

b) What causes the angular width of the Earth to appear to change?

- c) Instead of looking at planets, imagine you were to look at a car parked directly in front of you, and then at the same car parked on the far side of a large parking lot. In which location would the car appear larger?

- d) In c), does the actual size of the car change?

- e) In c), what changes? Would this change cause the angular width of the car to change?

6. Read the following dialogue between two students, and answer the associated questions below. Keep in mind that the apparent width is how big or small the object appears to be.

Student 1: “So I guess the apparent width of an object only depends on the actual size of an object.”

Student 2: “But I thought the apparent width of an object only depends on how far away you are from it—the distance between you and the object.”

- a) After reading the conversation between the two students, who do you think is right? (select 1)
 - i. Student 1
 - ii. Student 2
 - iii. Both students
 - iv. Neither student

- b) If one or both of the students are incorrect, modify their statements to make them both correct here.



Calculations

7. In step 2 you measured the angular widths of the planets at 3 different distances.
- a) Record those distances and the associated angular widths in fingers in the following data tables:

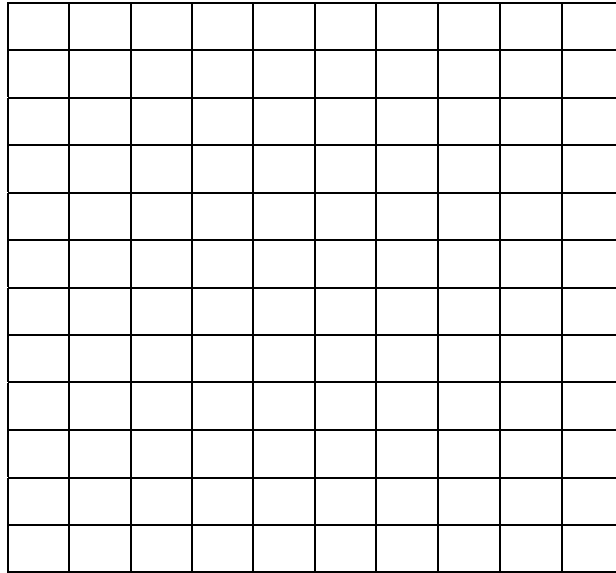
Pluto	
Distance (m)	Angular width (fingers)
1.	
2.	
3.	

Earth	
Distance (m)	Angular width (fingers)
1.	
2.	
3.	

- b) Plot the above data below, and draw a line through the data points:

Pluto' s Angular Width

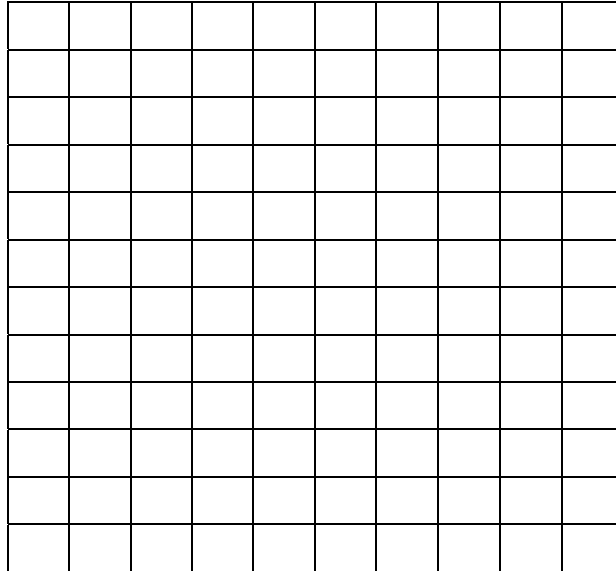
Angular width
(fingers)



Distance (meters)

Earth' s Angular Width

Angular width
(fingers)



Distance (meters)

c) Compare the two graphs you created. How are they similar?

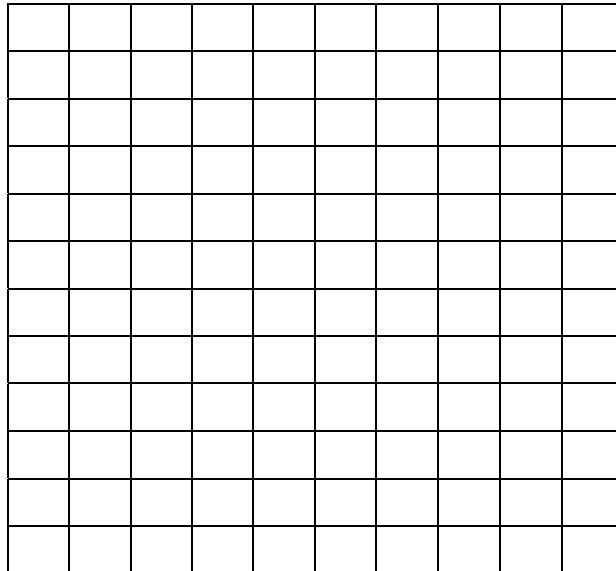
d) Describe the trend or pattern you observe in the graphs in your own words.

8. Refer to step 3. Record your values in the table, and plot the data in the graph provided.

Planet: _____	
Distance (m)	Angular width (number of fingers)
1.	1
2.	2
3.	3

Angular Width of Planet: _____

Angular width
(fingers)



Distance (meters)

a) Compare the graph here with the two graphs you created in number 8. What are the similarities and differences? Describe any relationship you observe between distance and angular width.

9. Refer back to step 1 (in Data Collection section) for the following values:

Record the distance you were standing away from the Degree Ruler here: _____

Record the angular width of your thumb in degrees in column two, below (in units of ‘degrees/thumb’).

Refer back to step 2 for the following values:

Record the angular widths of Pluto and Earth in ‘number of thumbs’ in column 1, below.

	Thumbs *	Degrees/thumb =	Degrees
Earth:			
Pluto:			

Calculate the number of degrees in the diagrams of Earth and Pluto based on your distance and the angular width of your thumb in column 3 above.

10. Throughout this exercise you have used the angular width in degrees of your finger, thumb and fist to determine the angular width of diagrams of planets. However, this tool is not limited to diagrams of planets in the classroom. You can use these values to observe the number of degrees between stars in the sky or in the angular width of the full moon!

HOMEWORK:

Extending your arm out straight in front of you, use the angular width of your finger in degrees to determine the angular width of the full moon in degrees. This is called the Moon's 'angular diameter'. How does the angular diameter relate to the Moon's actual diameter?

Answers to Student Data Sheet

Data Collection

Step 1: 4 m (unless you changed this distance); answers for angle measure will vary with the width of the students' fingers, thumbs, and fists and the length of their arms.

Step 2: Distances from planet diagrams: 1m, 2m, and 4m; answers will vary, but the angular width of Earth should be greater than Pluto at each distance

Step 3: Answers will vary

Observations

4. Values differ. The values are close, but some are slightly different. There is no one correct width or distance. Depends on the width of the finger, fist, thumb, and length of the arm.
 - a. Width of fingers and length of arms are different, not the size of the planet. The size of the planet does not change.
 - b. No, one is not wrong and the other right—they just have different sizes of fingers, thumbs and fists and different lengths of arms.
5. Row 1, distances: 1m, 2m, 4m
Row 2, angular widths: vary, but should be the same as in Step 2, above
 - a. Angular width of Earth decreases as you move farther away (distance increases).
 - b. Your distance from it causes the angular width of the Earth to appear to change.
 - c. If you are right next to the car it will appear larger.; The closer location.
 - d. No, the actual size of the car doesn't change.
 - e. The distance to the car changed. Yes, the change in distance does cause the angular width of the car to change, but not the actual width of the car.
6. The students statements are correct except for the use of the word “only”.
Apparent width depends on the actual size AND the distance from the object.
 - a. iv. Neither student is correct, since they both use the term “only” in their statements.
 - b. Student 1: The apparent width of an object depends on the actual size of an object and...
Student 2: the apparent width of an object depends on how far away you are from it—the distance between you and the object.

Calculations

7. In Step 2 they stood at 1m, 2m, and 4m and the widths will vary
 - a. Distances should be 1m, 2m, and 4m; widths will vary, but should match those from Step 2, above.
 - b. Generally, as distance increases the angular width should decrease
 - c. Slopes should be similar, but Pluto's will be lower on the y-axis because it has a smaller angular width compared with Earth at the same distances.
 - d. As the distance increases (further away from the diagrams) the angular width decreases.

8. In Step 3 they had to stand at a distance where the planet was 1 finger wide, 2 fingers wide, and 3 fingers wide. Then they measured the three distances. Answers should be the same as from Step 3, above.
 - Table: Distance 1 should be the greatest, and distance 3 should be the smallest.
 - Graph: y-axis values should be 1, 2, and 3. Distances should correspond, as in table above.
 - a. Trend should be the same as in the previous graphs. As the distance increases on the x-axis, the angular width or number of fingers decreases. (Inverse relationship between angular width and distance).
9. In Step 1, they stood at a distance of 4m (unless you changed that distance), and determined the angular width of their finger, thumb and fist using the degree ruler.
 - Distance: 4m
 - Angular width of their thumb in degrees from step 1 should be recorded in column 2 (both row 1 and row 2)
 - Then from Step 2, they must transfer the angular widths of Pluto and Earth in 'number of thumbs'.
 - Then they can multiply the number of thumbs by the degrees per thumb to get the degrees of the Earth and Pluto in this scale.

Extensions and Adaptations:

- If you do not have enough space for three stations, you can simply take down the Degree Ruler once all students have finished step 1, and replace it with a diagram of Earth and of Pluto side-by-side, although this will take longer.
- If you would like to use a distance other than 4 meters to calibrate students' fingers, thumbs, and fists as units of measurements, you will need to make a different Degree Ruler, as follows. Substitute your distance r , in meters, in place of 4 m to calculate the new centimeters per degree, following this example:
 - i. circumference, $c = 2 \pi r$
 - ii. since there are 360° in a circle, $c / 360^\circ =$ your increment in meters
 - iii. your increment in meters per degree * 100 = your increment in cm per degreeExample using $r = 4$ m:
 - i. $c = (2)(3.14\dots)(4 \text{ m}) = 25.1327 \text{ m}$
 - ii. $25.1327 \text{ m} / 360^\circ = 0.0698 \text{ m/degree}$
 - iii. $0.0698 \text{ m/degree} * 100 =$ about 7 cm in one degree
 - Then you would have to make a new ruler with your increment in centimeters between every line or degree, as in the Degree Ruler provided.
- Simplification for lower grades: To modify this activity for lower grade levels...
 - Divide class into two groups. Ask students to form two lines with the groups facing each other (like in 'red-rover'). Pairs are formed between the student in group 1 and the student in group 2 that are facing each other. Or students in both rows can count off, and like numbers become pairs.
 - One row remains stationary with the each student in the other row moves to a distance where his/her two thumbs—held at eye-level, with both arms fully extended and thumbs pressed together—just covers his/her partner's face. They note this distance using some unit of measurement, like number of tiles on the floor between them, number of footsteps, etc.
 - Then the same students must move to a distance where just one thumb covers his/her partner's face (with arm held at eye-level and one arm fully extended). They must note this distance using the same unit of measurement.
 - Roles reverse, and activity is repeated.
 - Discussion should include 'what changed'? Did the actual size of your partner's face change? After discussion make sure students understand that the distance changed and therefore the apparent size changed. Relate to common objects like trees that look small from far away, etc.
- Calculations for higher grades:

Students can calculate the small angle formula, relative intensity and the angle subtended by the solar disk from Earth and Pluto and compare. Here are several links that discuss these calculations:

<http://glast.sonoma.edu/teachers/agn/agnposactigpo.pdf>;

<http://astro.uchicago.edu/cara/outreach/resources/ysi94/smallangle.html>

<http://www.starhop.com/High/SolInt-19.pdf>

<http://www.marsacademy.com/amazsp/a4.htm>

- Adaptations for students with disabilities:
 - Seeing impaired: pair these students with other students whom are able to see. It may be useful to compare the size of an object to the sound of something familiar. For example, for a given sound, as the student moves away the sound will become fainter. Likewise, as the student moves away from an object its size will appear smaller, although neither the sound nor the size of an object has actually changed.
 - Hearing impaired: these students should be able to follow the activity by reading the Student Data Sheet as they proceed. Since measurements can be read or indicated using simple hand gestures, there need not be any special considerations for choosing partners with hearing impaired students.
 - Physically impaired: assuming the classroom has been arranged with adequate pathways and stations, students with physical impairments should have no problems completing this activity.

Interactive: with the link provided, see what the Sun looks like from Pluto using the **Solar System Simulator!**

- Navigate your browser here: <http://space.jpl.nasa.gov/>
- **Targets and Date**
 - Select the following options from the first two drop down menus: “The Sun” and “Pluto”
 - Choose today’s date or any other date you wish!
- **Field of View**
 - Choose the first round radio button, ‘I want a field of view of [x] degree(s)
 - Select the degrees from the drop down menu. Begin with 2 or 5 degrees, and then gradually enlarge the field of view.
- You may also wish to see what the Sun looks like from Earth for reference, since this will be quite familiar to students.

Standards Addressed:

Benchmarks (AAAS, 1993)

Chapter 2. The Nature of Mathematics

2C. Mathematical Inquiry

Grades 6 through 8

- Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves (for example, a proposition about "all equal-sided triangles" or "all odd numbers").

Chapter 9. The Mathematical World

9B: Symbolic Relationships

Grades 6 through 8

- Mathematical statements can be used to describe how one quantity changes when another changes. Rates of change can be computed from differences in magnitudes and vice versa.
- Graphs can show a variety of possible relationships between two variables. As one variable increases uniformly, the other may do one of the following: increase or decrease steadily, increase or decrease faster and faster, get closer and closer to some limiting value, reach some intermediate maximum or minimum, alternately increase and decrease indefinitely, increase or decrease in steps, or do something different from any of these.

Chapter 11. Common Themes

11B: Models

Grades 6 through 8

- Models are often used to think about processes that happen too slowly, too quickly, or on too small a scale to observe directly, or that are too vast to be changed deliberately, or that are potentially dangerous.

Chapter 12. Habits of Mind

12B: Computation and Estimation

Grades 6 through 8

- Estimate distances and travel times from maps and the actual size of objects from scale drawings.

12D: Communication Skills

Grades 6 through 8

- Organize information in simple tables and graphs and identify relationships they reveal.

National Science Education Standards (NRC, 1996)

Content Standards: 5-8

Science as Inquiry, CONTENT STANDARD A:

- Abilities necessary to do scientific inquiry

Earth and Space Science, CONTENT STANDARD D:

- Earth in the solar system

History and Nature of Science, CONTENT STANDARD G:

- Nature of science

Principles and Standards for School Mathematics (NCTM, 2000)

Measurement Standard

Grades 6 through 8

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements